

# Conduction-electron spin resonance in two-dimensional structures

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The influence of the conduction-electron spin magnetization density, induced in a two-dimensional electron layer by a microwave electromagnetic field, on the reflection and transmission of the field is considered. Because of the induced magnetization and electric current, both the electric and magnetic components of the field should have jumps on the layer. A way to match the waves on two sides of the layer, valid when the quasi-two-dimensional electron gas is in the one-mode state, is proposed. By following this way, the amplitudes of transmitted and reflected waves as well as the absorption coefficient are evaluated.

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Highlights:

1. Special matching conditions are needed to evaluate spin-resonance absorption of 2D conductor;
2. Magnetic field acting on electron spins differs from that of incident wave.

## I. INTRODUCTION

Electron spin resonance has long been used to determine  $g$  factors and the longitudinal and transversal relaxation times  $T_1$  and  $T_2$  providing information about electron band structure and allowing to investigate interactions responsible for spin-flip transitions [1]. This method has acquired an enhanced actuality nowadays because a growing interest in the spin dynamics in two-dimensional (2D) electron systems, which are potentially important for spintronics applications [2, 3]. For a long time it was thought that the direct observation of the conduction-electron spin resonance (CESR) in 2D structures is impossible because of

small number of current carriers. A breakthrough in this field are recent works [4–10] where the CESR in some 2D semiconductor structures was detected by means of the microwave absorption. The idealness and hence the conductivity of such structures can be high so that the field acting on electron spins can differ appreciably from that of incoming wave because of the field of the electric current excited by the wave. Despite the theory of the spin resonance excitation in bulk conductors is well elaborated (see, e.g., Refs. [11] and references therein), an analogous theory for 2D conductors, to the best of the author knowledge, is still lacking. The purpose of the present note is to fill in this gap.

## II. PROBLEM STATEMENT AND RESULTS

A feature of this problem which impedes the immediate application of standard methods, consists in the following. Let the quasi-2D layer aligned along an  $x - y$  plane is placed at position  $z = 0$  between two dielectrics with the permittivities  $\epsilon_1$  ( $z < 0$ ) and  $\epsilon_2$  ( $z > 0$ ), and  $z$ -axis points "upward" to the dielectric 2. Within the frame of classical electrodynamics, properties of a conducting medium enter the Maxwell equations through the material constitutive relations [12], which in the case under study have the form

$$\mathbf{J} = \hat{\sigma}\mathbf{E}, \quad \mathbf{M} = \hat{\chi}\mathbf{H}, \quad (1)$$

where  $\sigma$  and  $\chi$  are tensors of the electric conductivity and the magnetic susceptibility, respectively. In the following all quantities are assumed to have the time dependence  $e^{-i\omega t}$ . The great difference between the width of the conducting layer  $d$  and the wavelength  $\lambda = 2\pi/q_0$  ( $q_0 = \omega/c$ ) of microwave radiation urges one to treat the layer as strictly two-dimensional sheet so that  $\mathbf{J}(\mathbf{r}; t) = \delta(z)\mathbf{J}_S(x, y; t)$  and  $\mathbf{M}(\mathbf{r}; t) = \delta(z)\mathbf{M}_S(x, y; t)$ . Then, from the Amper law

$$\nabla \times \mathbf{H}_\omega = -iq_0\epsilon\mathbf{E}_\omega - \frac{4\pi}{c}\mathbf{J}_\omega, \quad (2)$$

it follows the usual expression for the jump of the magnetic field on the sheet

$$\hat{\mathbf{n}} \times (\mathbf{H}_{\omega,2} - \mathbf{H}_{\omega,1}) = \frac{4\pi}{c}\mathbf{J}_{\omega,S} \quad (3)$$

where  $\mathbf{n} = \hat{\mathbf{z}}$  and  $\mathbf{H}_{\omega,1}$  and  $\mathbf{H}_{\omega,2}$  are the values of the magnetic field on the lower and upper sides of the sheet, respectively. Accordingly, from the Faraday law

$$\nabla \times \mathbf{E}_\omega = iq_0 (\mathbf{H}_\omega + 4\pi\mathbf{M}_\omega) \quad (4)$$

it follows

$$\hat{\mathbf{n}} \times (\mathbf{E}_{\omega,2} - \mathbf{E}_{\omega,1}) = 4\pi i q_0 \mathbf{M}_{\omega,S\parallel}, \quad (5)$$

where  $\mathbf{M}_{\omega,S\parallel}$  is the parallel component of the 2D magnetization density. Thus both the jumps of the electric and magnetic components of the electromagnetic field on the sheet should be taken into account. If one tries, as usually, to utilize Eqs. (3) and (5) as boundary conditions for matching the fields above and below the sheet, an ambiguity occurs – the jumps make undefined the values of  $\mathbf{E}$  and  $\mathbf{H}$  which should be used in Eqs. (1). Thus, the inequality  $d \ll \lambda$  does not allow one to consider the system as a strictly 2D sheet from the very beginning. Therefore, we will first consider  $d$  as small but finite quantity, trying to find an additional property of the 2D conductor, which could lift the ambiguity mentioned, and take the limit  $d/\lambda \rightarrow 0$  on a later stage.

This additional property, which the following consideration depends on, is the assumption that the electron gas is in the one-mode state, i.e., all electrons occupy only the ground state in the confinement potential forming the 2D gas. Such a situation is usual in semiconductor heterostructures and conducting surfaces and interfaces of oxide insulators. It will be shown below that at the normal incidence of the wave on the one-mode gas the 'averaged' fields  $\mathbf{E}_{\omega,av} = \frac{1}{2}(\mathbf{E}_{\omega,1} + \mathbf{E}_{\omega,2})$  and  $\mathbf{H}_{\omega,av} = \frac{1}{2}(\mathbf{H}_{\omega,1} + \mathbf{H}_{\omega,2})$ , where  $\mathbf{E}_{\omega,1,2}$  ( $\mathbf{H}_{\omega,1,2}$ ) are the limit values of the electric (magnetic) field on the lower and upper sides of the layer, respectively, should be substituted into the right-hand sides of Eqs. (3) and (5). Thus, Eqs. (3) and (5) should take the form

$$\hat{\mathbf{n}} \times (\mathbf{H}_{\omega,2} - \mathbf{H}_{\omega,1}) = \frac{4\pi}{c} \hat{\sigma} \left( \frac{\mathbf{E}_{\omega,1} + \mathbf{E}_{\omega,2}}{2} \right), \quad (6)$$

$$\hat{\mathbf{n}} \times (\mathbf{E}_{\omega,2} - \mathbf{E}_{\omega,1}) = 4\pi i q_0 \hat{\chi} \left( \frac{\mathbf{H}_{\omega,1} + \mathbf{H}_{\omega,2}}{2} \right). \quad (7)$$

The standard method supplemented with this matching conditions becomes well defined and straightforwardly gives rise to the following results. The amplitudes of reflection  $T_{re}$  and

transmission  $T_{tr}$  have the form

$$\begin{aligned}
T_{re} &= \frac{N_{re}}{D}, T_{tr} = \frac{N_{tr}}{D}, \\
N_{re} &= \left( n_2 - n_1 + \frac{4\pi\sigma_\omega}{c} \right) + 2\pi i q_0 \chi_\omega \left[ 2n_1 n_2 + \frac{2\pi\sigma_\omega}{c} (n_1 - n_2) \right], \\
N_{tr} &= 2n_2 + 2\pi i q_0 \chi_\omega \frac{4\pi\sigma_\omega}{c} n_2 \\
D &= \left( n_2 + n_1 + \frac{4\pi\sigma_\omega}{c} \right) - 2\pi i q_0 \chi_\omega \left[ 2n_1 n_2 + \frac{2\pi\sigma_\omega}{c} (n_1 + n_2) \right],
\end{aligned} \tag{8}$$

while the absorption coefficient, with the accuracy up to terms linear in  $\chi$ , is

$$\begin{aligned}
A &= \frac{N}{Z}, \\
N &= \frac{4\pi\sigma'_\omega}{c} n_1 + 4\pi q_0 n_1 n_2^2 \chi''_\omega \left[ 1 + \frac{16\pi\sigma'_\omega}{c n_2} + \left( \frac{2\pi\sigma'_\omega}{c n_2} \right)^2 + \left( \frac{2\pi\sigma''_\omega}{c n_2} \right)^2 \right], \\
Z &= \left( n_1 + n_2 + \frac{2\pi\sigma'_\omega}{c} \right)^2 + \left( \frac{2\pi\sigma''_\omega}{c} \right)^2.
\end{aligned} \tag{9}$$

These equations have been written for the fields with the circular polarization  $\mathbf{e}_+ = \frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y)$  when  $\mathbf{E} = E_{(-)}\mathbf{e}_+$ ,  $\mathbf{H} = H_{(-)}\mathbf{e}_+$ ,  $\mathbf{M} = M_{(-)}\mathbf{e}_+$ ,  $\mathbf{J} = J_{(-)}\mathbf{e}_+$  and the constitutive relations have the form  $M_{(-)} = \chi_\omega^{(+)} H_{(-)av}$  with  $\chi_\omega^{(+)} = \chi_{xx}(\omega) + i\chi_{xy}(\omega)$  and  $J_{(-)} = \sigma_\omega^{(+)} E_{(-)av}$  with  $\sigma_\omega^{(+)} = \sigma_{xx}(\omega) + i\sigma_{xy}(\omega)$ . Also the following notations have been used:  $n_{1,2} = \sqrt{\epsilon_{1,2}}$  is the refraction index,  $\chi''_\omega = \Im \chi_\omega^{(+)}$ ,  $\sigma_\omega = \sigma_\omega^{(+)}$ ,  $\sigma'_\omega = \Re \sigma_\omega^{(+)}$ , and  $\sigma''_\omega = \Im \sigma_\omega^{(+)}$ . Near the frequency  $\omega_{res}$  of the CESR one gets [13]  $\chi_\omega^{(+)} \cong \chi_0 \frac{\pi}{m} N(\epsilon_F) \frac{-\omega}{\omega - \omega_{res} + \frac{i}{T_2}}$ , where  $\chi_0 = \frac{m}{\pi} \left( \frac{g\mu_B}{2} \right)^2$  is the static susceptibility of 2D degenerate electron gas and  $N(\epsilon_F)$  is the density of states for a single spin. Eqs. (8) and (9) show that at  $\sigma/c \geq 1$  the effect of the electric current, induced by microwave field, on effective magnetic field acting on electron spins can be appreciable. The derivation of Eqs. (8) and (9) is quite standard and therefore is not given here. The remaining part of the paper presents the proof of the above matching conditions.

### III. MATCHING CONDITIONS

So we consider the electron gas which occupies the layer  $-\frac{d}{2} \leq z \leq \frac{d}{2}$ . Two facts follow from the assumption about the one-mode state of the gas (see Appendix). The first is that the coordinate dependence of the 3D density of the current and the magnetization has the

factorized form

$$\mathbf{J}(\mathbf{r}, t) = \rho(z)\mathbf{J}_S(\mathbf{r}_\parallel, t), \quad \mathbf{M}(\mathbf{r}, t) = \rho(z)\mathbf{M}_S(\mathbf{r}_\parallel, t), \quad (10)$$

where  $\mathbf{r} = (x, y, z) = (\mathbf{r}_\parallel, z)$ ,  $\rho(z) = |\psi_0(z)|^2$ ,  $\psi_0(z)$  is the wave-function of the ground state, and  $\mathbf{J}_S$  and  $\mathbf{M}_S$  are the 2D densities. At the normal incidence of the radiation,  $\mathbf{J}_S$  and  $\mathbf{M}_S$  lose their coordinate dependence. Second fact is that the constitutive relations (1) take the form

$$J_S^i(\omega) = \sigma_\omega^{ij} \int_z \rho(z) E^j(z, \omega), \quad M_S^i(\omega) = \chi_\omega^{ij} \int_z \rho(z) H^j(z, \omega), \quad (11)$$

where  $\int_z = \int dz$ .

Consider first the question about the value of the electric field which should be used in the Ohm's law in the limit  $d/\lambda \rightarrow 0$ . As it is known [and also seen from Eq. (5)], the major reason for a finite difference between the electric field on the upper and lower surfaces of the layer is the magnetization. To make the following explanation more clear the effect of the external magnetic field is omitted for a while. Consider a strictly 2D sheet, uniformly filled with the spin magnetization  $\rho(\zeta)\mathbf{m}^{-i\omega t}$ ,  $\mathbf{m} \perp \mathbf{e}_z$  [ $\mathbf{m}$  does not depend on  $\mathbf{r}_\parallel$  at the normal incidence], which lies inside the layer at  $z = \zeta$ ,  $|\zeta| \leq \frac{d}{2}$ . By utilizing the fact that the vector-potential created at the point  $\mathbf{r}$  by the magnetic dipole  $\boldsymbol{\mu}(\mathbf{r}_0)$  placed at the point  $\mathbf{r}_0$  is given by  $\mathbf{A}(\mathbf{r}) = \frac{\boldsymbol{\mu}(\mathbf{r}_0) \times (\mathbf{r} - \mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|^3}$  [14], one can show that the vector-potential created by the magnetization of the sheet is

$$\mathbf{A}(\mathbf{r}, t) = e^{-i\omega t} 2\pi \rho(\zeta) (\mathbf{m} \times \mathbf{e}_z) \text{sign}(z - \zeta), \quad (12)$$

so that the vector-potential created by the total magnetization of the electron layer is given by

$$\mathbf{A}_\omega(z) = 2\pi (\mathbf{m} \times \mathbf{e}_z) \left[ \int_{-d/2}^z \rho(\zeta) d\zeta - \int_z^{d/2} \rho(\zeta) d\zeta \right]. \quad (13)$$

The corresponding part of the electric field,  $\mathbf{E}_\omega = \frac{i\omega}{c} \mathbf{A}_\omega$ , has the same space dependence. According to Eq. (11), the electric current induced by *this* part of the field is defined by the expression

$$\int_{-d/2}^{d/2} \rho(z) \mathbf{E}_\omega(z) dz \sim (\mathbf{m} \times \mathbf{e}_z) \int_{-d/2}^{d/2} \rho(z) \left[ \int_{-d/2}^z \rho(\zeta) d\zeta - \int_z^{d/2} \rho(\zeta) d\zeta \right] dz, \quad (14)$$

which equals zero at any function  $\rho(z)$ . To see this fact one should consider the second term in Eq.(14) as the double integral  $\int \int dz d\zeta \rho(z) \rho(\zeta)$  over the triangle region  $-\frac{d}{2} \leq$

$z, \zeta \leq \frac{d}{2}$ ,  $z \leq \zeta$ , perform the  $z$ -integration first, and then change variables  $z \leftrightarrow \zeta$ . Thus, in the case of one-mode conductor, when the space dependence of both the current and the magnetization densities are defined by the same function  $\rho(z)$ , that part of the electric field, which is induced by the oscillating magnetization, does not give rise to the electric current. This result is the central point in the derivation of Eqs. (6,7). Within the layer, the total electric field  $\mathbf{E}_\omega(z)$  can be represented as the sum of a slow-space-varying component  $\mathbf{E}_{sl}(z)$ , whose scale of variance is the wave length  $\lambda$ , and the fast-space-varying component  $\mathbf{E}_f(z)$  due to the magnetization [which yields the finite jump on the layer in the limit  $d/\lambda \rightarrow 0$ ], as  $\mathbf{E}_\omega(z) = \mathbf{E}_{sl}(z) + \mathbf{E}_f(z)$ . The fast component does not participate in the Ohm's law while the slow component is almost constant within the layer and with the accuracy up to corrections of the order of  $d/\lambda$  can be taken at any point inside the layer, say at  $z_0$ ,  $|z_0| < \frac{d}{2}$ . But, with the same accuracy, we have  $\mathbf{E}_{sl}(z_0) = \frac{1}{2} [\mathbf{E}_{sl}(\frac{d}{2}) + \mathbf{E}_{sl}(\frac{-d}{2})]$ . Then, since  $\int_z \rho(z) = 1$ , we came to the Ohm's law with the ansatz  $\mathbf{E}_\omega = \mathbf{E}_{sl,av}$ . But because for the fast component one has  $\mathbf{E}_f(\frac{d}{2}) + \mathbf{E}_f(\frac{-d}{2}) = 0$ , one may change the 'average' of the *slow* field  $\mathbf{E}_{sl,av}$  by the 'average' of the *total* field  $\mathbf{E}_{\omega,av}$ . After then one can take the limit  $d/\lambda \rightarrow 0$  thereby obtaining Eq. (6).

The Eq. (7) can be proved following the same lines: by using the fact that the magnetic field created at the point  $\mathbf{r}$  by the current with the density  $\mathbf{j}(\mathbf{r}_0)$  is given by  $\mathbf{H}(\mathbf{r}) = \frac{1}{c} \int_{\mathbf{r}_0} \frac{\mathbf{j}(\mathbf{r}_0) \times (\mathbf{r} - \mathbf{r}_0)}{|\mathbf{r} - \mathbf{r}_0|^3}$  [14], one can find explicitly the magnetic field inside the layer induced by the electric current and to show that this magnetic field does not give rise to a contribution to the magnetization. Since at the presence of an applied constant magnetic field the form of Maxwell's equations for  $\mathbf{e}_+$  and  $\mathbf{e}_-$  circular polarizations coincides with that at the absence of the field, the proof presented holds in those cases as well.

#### IV. CONCLUSIONS

In summary, it has been considered one problem of the macroscopic electrodynamics of 2D paramagnetic conductors, in which it is necessary to take care of jumps of both the electric and magnetic component of the electromagnetic field on the conductor. Namely, the reflection of electromagnetic wave with a frequency near the CESR. Some way to solve this problem has been pointed out. Physical sense of the matching conditions found, Eqs. (6) and (7), is that if the electron gas is in the one-mode state one can disregard the jump of

the electric field by evaluating the jump of the magnetic field and, quite analogously, one can disregard the jump of the magnetic field by evaluating the jump of the electric field. The results obtained, Eqs. (8) and (9), describe the contribution of the CESR, as well as the cyclotron resonance, to the transmission/reflection amplitudes and to the absorption coefficient. In the limit of the very small conductivity, Eqs. (8,9) describe the transmission through a paramagnetic insulator, while at  $\chi_\omega \rightarrow 0$  we recover the absorption only due to the cyclotron resonance [15]. Note that the statement of the problem of the CESR excitation adopted in this paper holds for 2D structures with small up-down asymmetry, like Si/SiGe quantum wells investigated in works [4–6, 8–10]. In such structures the ESR reveals itself in the ordinary fashion - through  $\chi_\omega$ . In semiconductor structures with strong Rashba spin-orbit coupling, the CESR can reveal itself more pronouncedly through  $\sigma_\omega$  than through  $\chi_\omega$ . This is the case, e.g., in AlAs quantum wells, as it has been shown experimentally [7] and theoretically [13]. In those cases, the spin susceptibility (and hence the jump of the electric field on the layer) plays a minor role and can be disregarded.

Note that special matching conditions discussed above are not peculiar to electrodynamics of conventional 2D systems. They are also required for an evaluation of the paramagnetic response of 2D conductors with the Weil-type Hamiltonian.

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## Appendix A

In this Appendix it is shown how one can derive Eqs. (10) and (11). A straightforward way is to use the Kubo formalism [16] which yields, for example, for the 3D current density

$$J_i(\mathbf{r}|\omega) \sim \int_{\mathbf{r}'} \Pi_{ij}^R(\mathbf{r}, \mathbf{r}'|\omega) E_j(\mathbf{r}'|\omega), \quad (\text{A1})$$

where  $\Pi_{ij}^R(\mathbf{r}, \mathbf{r}'|\omega)$  is the retarded velocity correlation function, which in the Feynman diagram language is given by a sum of loop ladder diagrams. For the conducting layer in the one-mode state and at the normal incidence of the radiation, when the electric field is parallel to the layer and is a function of only  $z$ , the radiation cannot induce transitions into excited states in the confinement potential. Under these conditions, the 3D one-electron Green's function  $G^{3D}$  relevant to the problem can be expressed through the 2D Green's

function  $G_S$  as

$$G^{3D}(\mathbf{r}, \mathbf{r}'|\epsilon) = \psi_0(z)G_S(\mathbf{r}_\parallel, \mathbf{r}'_\parallel|\epsilon)\psi_0(z'). \quad (\text{A2})$$

Consider the contribution to  $\Pi_{ij}^R$  of the simplest diagram (which is the loop without impurity insertions). Since the parallel components of the velocity operator  $\hat{\mathbf{v}}(\mathbf{r}_\parallel)$  do not act on the wave functions of perpendicular motion  $\psi_0(z)$  and  $\psi_0(z')$ , this diagram yields

$$T \sum_{\epsilon} |\psi_0(z)|^2 \text{Tr} \{ \hat{v}_i(\mathbf{r}_\parallel) G_S^R(\mathbf{r}_\parallel, \mathbf{r}'_\parallel|\epsilon + \omega) \hat{v}_j(\mathbf{r}'_\parallel) G_S^A(\mathbf{r}'_\parallel, \mathbf{r}_\parallel|\epsilon) \} |\psi_0(z')|^2 \quad (\text{A3})$$

so that the corresponding contribution to the current  $J_j(\mathbf{r}|\omega)$  is proportional to

$$|\psi_0(z)|^2 \int_{\mathbf{r}'_\parallel, z'} T \sum_{\epsilon} \text{Tr} \{ \hat{v}_i(\mathbf{r}_\parallel) G_S^R(\mathbf{r}_\parallel, \mathbf{r}'_\parallel|\epsilon + \omega) \hat{v}_j(\mathbf{r}'_\parallel) G_S^A(\mathbf{r}'_\parallel, \mathbf{r}_\parallel|\epsilon) \} |\psi_0(z')|^2 E_j(z'|\omega) \quad (\text{A4})$$

It is seen that this expression reproduces the form of Eqs. (10) and (11). It can be straightforwardly checked that the same is true with respect to contributions of all other diagrams. The validity of the 'magnetic' parts of Eqs. (10) and (11) can be proved quite analogously.

$$\mathbf{J}(\mathbf{r}, t) = e\mathbf{v}(t)|\psi(\mathbf{r} - \mathbf{R}(t))|^2 \quad (\text{A5})$$

$$\mathbf{F} = \int e|\psi(\mathbf{r})|^2 \mathbf{E}(\mathbf{r}) d^3\mathbf{r} \quad (\text{A6})$$

$$\mathbf{J}_S(\omega) = \sigma_\omega \int_z \rho(z) \mathbf{E}(z, \omega), \quad \mathbf{M}_S(\omega) = \chi_\omega \int_z \rho(z) \mathbf{H}(z, \omega), \quad (\text{A7})$$

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